
REPORT No. 175.

**ANALYSIS OF W. F. DURAND'S AND E. P. LESLEY'S
PROPELLER TESTS**

By MAX M. MUNK

National Advisory Committee for Aeronautics

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SUMMARY.

The following paper, prepared for the National Advisory Committee for Aeronautics, is a critical study of the results of propeller model tests with the view of obtaining a clear insight into the mechanism of the propeller action and of examining the soundness of the physical explanation generally given. The nominal slipstream velocity is plotted against the propeller tip velocity, both measured by the velocity of flight as a unit. Within the range corresponding to conditions of flight, the curve thus obtained is a straight line. Its inclination depends chiefly on the effective blade width, its position on the effective pitch. These two quantities can therefore be determined from the result of each propeller test. Both can easily be estimated therefrom for new propellers of similar type. Thus, a simple method for the computation of propellers suggests itself.

The slip curve mentioned is not a straight line along its entire length. At a small relative tip velocity it is bent up, because the lift curve of the blade sections used is bent up that way at small lift coefficients. At a certain high relative tip velocity the slip curve shows a break and runs then straight again but at a different slope. The slope is increased so that at zero advance the propeller develops a larger thrust than could be expected from the magnitude of the thrust in flight.

REFERENCES.

- (1) William F. Durand and E. P. Lesley, Experimental Research on Air Propellers, Technical Reports, N. A. C. A. 14, 30, 64, 109 and 141.
- (2) Max M. Munk, Notes on Propeller Design, Technical Notes, N. A. C. A. 91 and 94.

INTRODUCTION.

There is at present still some controversy as to the exact explanation of propeller action, an important problem, because the successful computation and design of propellers depend on its solution. The present paper is an attempt to approach the solution by means of analyzing a large and systematic series of propeller model tests, examining the uniformity and regularity of the results, and establishing then the general laws underlying them.

Dr. W. F. Durand's and E. P. Lesley's series of propeller tests (ref. 1) is the most perfect and complete one ever published. The tests are selected and executed in the most careful way, the method of conducting them is excellent, and the type of wind tunnel is chosen most suitably for this purpose. It seems therefore most expedient to begin with an analysis of these tests. This paper is confined to them only. It is, however, the intention of the author later to extend the results by a similar analysis of other propeller tests available.

ACTION OF THE BLADE ELEMENTS.

The general principles of propeller action are generally agreed upon in so far as the blade elements are understood to act like portions of wings. Hence a regular and uniform relation between the propeller forces and the characteristics of its motion can not be expected, except if the blade sections themselves show a regular aerodynamic characteristic as wings. The blade sections of the propellers under consideration are not perfectly ideal in this respect.

Especially at small angles of attack, the sections are likely to produce a lift of irregular magnitude. At larger angles of attack, however, the lift curve is more than likely to run straight and to have a slope at least close to the one shown by ideal sections. It is, therefore, to be expected that over a considerable range the propellers will give regular results as far as this regularity is dependent on that of the blade section action.

THE SLIPSTREAM.

In addition, propeller action is determined by the general characteristic of the air flow created. The propeller, moving with the velocity of flight V through air originally at rest, leaves behind it a slipstream whose final average velocity may be denoted by v . The propeller is in a region of air which has already assumed a part of this final slipstream velocity. There is some controversy as to how much. Imagine the propeller not progressing and the air moving with the velocity V as in a wind tunnel. Its increase of velocity when passing the propeller may be denoted by w , so that it passes with the velocity $w + V$. The thrust created per unit of mass of the passing air is equal to its final increase of velocity, that is to v . This thrust acts on air passing with the velocity $(w + V)$, and hence imparts to it the energy $(w + V)v$ per unit

of mass. This is equal to the increase of its kinetic energy which is $\left(\frac{v}{2} + V\right)v$ per unit of mass. Hence

$$(1) \quad w = \frac{v}{2}$$

It results, therefore, that the air when passing the propeller has already assumed just half of the slipstream velocity. Hence the mass passing the propeller disc per unit of time is

$$D^2 \frac{\pi}{4} \left(V + \frac{v}{2} \right) \rho$$

where D denotes the propeller diameter and ρ the density of the air. The thrust can therefore be expressed

$$(2) \quad T = D^2 \frac{\pi}{4} \left(V + \frac{v}{2} \right) v \rho$$

or otherwise written

$$(3) \quad \frac{T}{D^2 \frac{\pi}{4} V^2 \rho} = \left(1 + \frac{v}{V} \right)^2 - 1$$

The lefthand side of (3) represents a thrust coefficient, the thrust divided by the propeller disk area and divided by the dynamic pressure of the velocity of flight. It may be denoted by

$$C_T = \frac{T}{D^2 \frac{\pi}{4} V^2 \rho} \quad (\text{definition})$$

From equation (3) follows then

$$(4) \quad \frac{v}{V} = \sqrt{1 + C_T} - 1; \quad C_T = \left(1 + \frac{v}{V} \right)^2 - 1$$

For very small thrust coefficients C_T and relative slipstream velocities $\frac{v}{V}$, but only for such, equation (4) can approximately be written

$$(4a) \quad \frac{v}{V} = \frac{1}{2} C_T; \quad C_T = \frac{2v}{V}$$

This is the essential content of Froude's momentum theory of the propeller. Since the useful work of the propeller thrust is done at a velocity V , but the propeller acts in a region of air

passing it with the velocity $\left(V + \frac{v}{2}\right)$ there is a loss. Only the fraction $\frac{V}{V + \frac{v}{2}}$ of the horsepower

delivered to the propeller is reproduced by the propeller as thrust horsepower, the remaining part is used for the creation of the slipstream. There are other additional losses. The slipstream velocity is not quite uniform, and the air receives a small rotational velocity too, both of which results in a small increase of the slipstream loss. Then there is the friction of the air passing the propeller blades, quite a considerable item, since the blade velocity is rather large. Therefore

$$\eta_0 = \frac{V}{V + \frac{v}{2}} = \frac{1}{1 + \frac{1}{2} \frac{v}{V}}$$

is only the upper limit of the propeller efficiency, which a propeller will never actually reach; the efficiency is always smaller. This question is discussed in reference (2), and I am also taking it up in a later part of this paper.

THE SLIP CURVE.

The momentum theory, in particular equation (4), states that there is a simple relation between the thrust coefficient C_T and the average or—as I prefer to call it—nominal relative slipstream velocity v/V . Each of them can easily be converted into the other by use of the slide ruler. It is certainly more natural and convenient to use the thrust coefficient if the magnitude of the thrust itself is under consideration. Still, in the earlier parts of propeller computations, there are great advantages connected with the use of the relative slipstream velocity v/V instead of the thrust coefficient itself, which quantity is then finally converted into the thrust coefficient by means of equation (4). These advantages become particularly conspicuous if results of propeller tests are to be laid down or to be analyzed, as in the present paper. These advantages are the natural consequence of the fact that the conditions under which one particular propeller is working are primarily determined by the magnitude of two velocities; for instance, the velocity of flight V and the tangential component of the velocity of the propeller tip U

$$(5) \quad U = \pi n D$$

(where n denotes the number of revolutions per unit of time). It is probable in itself that a third and fundamental velocity, chosen in this paper the nominal slipstream velocity v , stands in a simpler relation to two other velocities than does a force. A closer examination confirms this. If (a) the blade sections acted like ideal wing sections in an ideal fluid and (b) the change of the shape of the slipstream had no influence on the air forces, the air flow being irrotational in all points other than the boundaries of the slipstream, the velocity at any point, however situated relative to the rotating propeller, would be a linear function of the two velocities describing the propeller motion; i. e.,

$$V' = AV + BU$$

where A and B are constants. Each of the velocities V or U , if existing alone, would create at the point a velocity AV or BU , respectively, proportional to the magnitude of the creating velocity V or U ; and if V and U are finite at the same time, the resulting distribution of velocity would be the superposition of the two flows created by each, giving rise to the last equation. The nominal slipstream velocity v is the average of many such velocities V' , and hence it too could be represented in the form

$$(6) \quad v = AV + BU$$

It is convenient to divide (6) by one of the propeller velocities V or U , particularly to divide it by V , the velocity of flight, although then the resulting equations need a special interpretation for the case of advance zero, $V=0$. The division gives

$$(6a) \quad \frac{v}{V} = A + B \frac{U}{V}$$

or otherwise written

$$(7) \quad \frac{v}{V} = m \left(\frac{U}{V} - \left(\frac{U}{V} \right)_0 \right)$$

$\left(\frac{U}{V} \right)_0$ is then that magnitude of the relative tip velocity U/V at which the slipstream velocity, and hence the thrust, becomes zero. It stands in a simple relation to the effective pitch ratio of the propeller, as will be shown later. Equation (7), expressing a relation between the two variables $\frac{v}{V}$ and $\frac{U}{V}$, is linear. Hence the relative slip velocity $\frac{v}{V}$ plotted against the relative tip velocity $\frac{U}{V}$ would be a straight line. I intend to make an extensive use of this curve, and it is therefore convenient to have a designation for it. I call it the "slip curve" in this paper. The magnitude of m indicates the slope of this slip curve and may be named the slip modulus of the propeller. Its discussion will be taken up later. All three assumptions made, the idea, blade sections, the ideal fluid, and the unchangeable shape of the slipstream are not strictly fulfilled. However, under ordinary conditions of flight, the first two can be accepted, as the air forces of the blade sections are in close agreement to those deduced therefrom. The shape of the slipstream boundary, too, is not very changeable, and its changes might not bring about serious changes of the air forces produced. This can not be settled definitely by discussion. It has to be left to tests to find out what really happens, which influences are the important ones, which assumptions and arguments are sound and which are not. The discussion gives a suggestion as to how to proceed. Actually plotting the slip curve v/V and U/V is a most convenient method for examining what happens. The shape of the curve thus found is a criterion for the expediency of the assumptions mentioned. If the slip curve appears to be a straight line, the propeller action can be interpreted and understood by the comparison with the ideal propeller acting in the ideal fluid neglecting the change of the slipstream shape. A practical method for computing propellers then readily suggests itself. If the slip curve does not appear to be straight, two explanations can be offered. Either the blade sections do not possess a regular aerodynamic characteristic under the conditions tested, or the distribution of the slipstream velocity is fundamentally different from the ideal constant velocity, and the boundary is modified, too, if one can speak of a boundary at all.

DISCUSSION OF THE SLIP CURVES FOUND.

We are now prepared to take the actual slip curves into view as they are computed from Doctor Durand's tests and plotted in the diagrams of this paper. As a rule, all slip curves of propellers only differing by the magnitude of the pitch are drawn together in one diagram. The first two pages, 16 figures containing 67 propellers, give the systematic series of Doctor Durand. Six additional diagrams show the slip curves computed from those tests of the same investigations that form small series in themselves and are likely to throw more light on the present problem.

In all diagrams, the relative tip velocity, $\frac{U}{V} = nD \frac{\pi}{V}$ is plotted horizontally, and the relative slipstream velocity, $\frac{v}{V} = \sqrt{1 + \frac{T}{\left(D^2 \frac{\pi}{4}\right) \left(V^2 \frac{\rho}{2}\right)}} - 1$ is plotted vertically. The portion of the curve

below the zero axis corresponds to negative thrust. This portion is not of interest for the practical use of the propeller, but the shape of the curve there gives an explanation for the shape of the slip curve for propellers of very small pitch (as, for instance, propeller No. 144) at moderate positive relative slip velocity. The slip curve for negative thrust is not at all straight, nor the slip curve for small positive thrust of propellers having a small pitch. There can hardly be any doubt about the reason for this. The angles of attack of the blade sections are then very small or negative and the sections have an irregular aerodynamic characteristic, which is reproduced in the shape of the slip curve.

The slip curve for propellers of normal pitch and positive and moderate relative slipstream velocity can be represented as a straight line in all cases. The observed points often lie very exactly on a straight line, as, for instance, with propeller No. 39. It is to be remembered that these tests are necessarily not very exact. It was also sometimes difficult to obtain the exact quantities as originally measured from the diagrams by which they are represented. $\frac{U}{V}$ in particular is to be computed by using the inverse ratio of $\frac{VN}{D}$, given in the diagram. This leads to large errors for very small values of $\frac{VN}{D}$, which is with all points far to the right and way up in the slip curves. All the highest points are unreliable for this reason.

It appears then from Doctor Durand's tests that within a considerable range and within the one of practical application the slip curve is a straight line. In the next section I proceed to examine whether the characteristics of these straight lines, their position and their slope, can be explained by comparison with the ideal propeller. It will appear that this can be done and the use of the slip curve furnishes therefore a good method for the computation of propellers. Before discussing that I wish to finish the discussion of the slip curves obtained from the tests.

The reader will notice that the curves deviate from a straight line not only at the lower ends, mostly at negative thrust, but at the upper ends too, though here in quite another way, there are irregularities with practically all slip curves. The slip curve has a break there and then follows a straight line again, but one with a steeper slope than underneath the break. The upper portions correspond to a small advance of the propeller. Then the thrust appears larger than would be expected from the lower portion of the slip curve.

These breaks are probably not to be explained by irregularities of the action of the blade section. In general, the break occurs at a positive, high relative tip velocity if the pitch is small. The last diagram, for a propeller with changeable pitch, is interesting. The breaks are here on a very regular curve. The tests do not give enough information to establish the reason for the breaks definitely. At horizontal flight the propellers are probably always in the range below the break, it is therefore not of paramount importance to study the reason of the breaks in detail. Still it is of interest. The occurrence of these breaks shows, for instance, how unreliable are the conclusions as to conditions of flight when drawn from tests at zero advance. The starting thrust is increased, due to the break, which fact in itself is quite desirable.

THE EFFECTIVE PITCH OBSERVED.

It is convenient to use the slip curve for the analysis and the computation of propellers, because this curve, being a straight line, is determined by two constants only, its position and its slope. The position is determined by its intersection with the horizontal zero axis at a point which was denoted by $\left(\frac{U}{V}\right)_0$. This is the relative tip velocity for zero thrust. If the blades were parts of mathematical helical surfaces with the pitch p and moving in a fluid without viscosity, this relative tip velocity would be

$$(8) \quad \left(\frac{U}{V}\right)_0 = D \frac{\pi}{p}$$

For at that relative tip velocity the air would be at rest at all points and the propeller would screw itself through it, as through a solid nut, experiencing no air forces whatever. A small viscosity would give rise to small tangential forces, producing no considerable thrust in itself. They would give rise to a small rotation of the air passing the propeller and thus indirectly originate some minor air forces and maybe a small thrust. This would hardly amount to much under ordinary conditions.

With actual propellers equation (8) does not hold true for another reason. This is the shape of the blade section. The propeller by no means has the shape of the helical surface or of a part of it, the pitch of which is used as nominal pitch. The blade section is ordinarily plane at the bottom and cambered at the top and the nominal pitch is that of a helical surface determined by the bottom. Now, as is well known from the study of ordinary wings, the direction of flight of such wings giving the lift zero is by no means parallel to its lower surface. If, however, the angle of attack is zero, the lift has a considerable positive value and the wing has to be turned back by 3° or more, depending on the shape of its section, in order to create no lift. From this experience it follows that the effective pitch of a propeller may be expected to be larger than the nominal pitch measured in the usual way from the lower surface of the blade.

The sections used in the tests of Doctor Durand's propeller series are not exactly the ones used in practice. The tests, therefore, can not be used for the study of the magnitude of the main constants of actual propellers, their effective pitch, and the effective blade width. The great value of Doctor Durand's tests is the convenient way in which they can be used to establish the general laws underlying propeller action. With this purpose in view it will be instructive and important to compute the average value of the angle of attack for the lift zero of the blade sections for one typical propeller test, and to compare this angle with that which may be reasonably expected from the study of ordinary wings. Since all propeller tests under consideration at present give results consistent with each other, this one trial will be sufficient to decide whether the blade section effect is really the explanation for the discrepancy between the nominal pitch and the effective pitch.

Take for instance propeller No. 20. The slip curve intersects with the horizontal zero axis at the point $\left(\frac{U}{V}\right)_0 = 3.75$. The nominal pitch ratio $\frac{P}{D} = 0.70$. For rough calculations the propeller blade can be supposed to be concentrated at a mean radius, say, at 0.70 of the radius. This gives a tangential velocity of 0.70 of the tip velocity, and the average relative tangential velocity for the thrust zero is therefore $3.75 \times 0.70 = 2.62$. This is the cotangent of $20^\circ 50'$. The nominal pitch is $0.7 D = 0.318 \times 0.70 D\pi$. 0.318 is the tangent of $17^\circ 40'$. The difference is $3^\circ 10'$. Add to this a small correction due to frictional drag, rotation of the slipstream and the curvature of the relative path between the air and the blade, of say, $\frac{1}{2}^\circ$ giving in all $3\frac{1}{2}^\circ$. This angle of zero lift is now indeed most plausible for the average blade section used. Thus this point is settled. The same computation should be made and the actual zero angle computed from tests with actual propellers or models thereof. It can then be assumed that propellers of similar type have the same zero angle and the effective pitch can easily be computed from it and the nominal pitch. The computation is to be made backward. (a) The effective pitch at a mean radius (say $0.7 r$) is converted into degrees. (b) The zero angle is added to it. (c) The sum is converted again into the pitch as ordinarily measured.

THE SLIP MODULUS.

The slip curves derived from the tests are nearly parallel for propellers only differing by the pitch. The curves have a larger slope if the mean blade width is larger. It would therefore appear that the slip modulus m depends chiefly on the mean relative blade width and is not very much influenced by the pitch.

It can be shown easily that such a law can be expected from a propeller with narrow blades, ideal blade sections, and low pitch ratio. With such propellers the influence of the slipstream velocity on the effective angle of attack can be neglected. Suppose the blade area S to be concentrated at the distance $0.7 r$ from the axis. The tangential velocity U' at this

point may be such as to give zero thrust at the velocity of flight V . Hence it corresponds to the intersection of the slip curve and the horizontal axis. In order to compute the slope of this curve, I proceed to compute a point slightly above the horizontal axis. The velocity of flight may remain V as before, but the tangential velocity may increase from U' to $U' + dU'$ where dU' is a differential. Then the cotangent of the angle of relative motion between the blade and the air is increased from U'/V to $(U' + dU')/V$ and hence the angle itself is increased by

$$d\alpha = \frac{dU'}{V} \frac{1}{\left(1 + \left(\frac{U'}{V}\right)^2\right)}$$

The square of the relative velocity between blade and air is

$$V^2 \left(1 + \left(\frac{U'}{V}\right)^2\right)$$

Hence the lift produced, being approximately equal to the thrust, is

$$T = 2\pi S V^2 \frac{\rho}{2} \frac{dU'}{V}$$

and the thrust coefficient is

$$C_T = \frac{8S}{D^2} \frac{dU'}{V}$$

equal to $2 \frac{v}{V}$ according to equation (4a). Hence

$$m = \frac{\frac{v}{V} 0.7}{\frac{dU'}{dV}} = \frac{4S}{D^2} 0.7$$

Actual propellers can not be considered as having an infinitely small blade width. If the tip velocity is increased, a finite slip velocity one-half m times as great as it, is produced, which neutralizes a part of the increase of the angle of attack of the blade, so that a smaller lift and slipstream velocity is produced and the slip modulus becomes smaller than according to the last formula. The modulus is not quite independent of the blade width, therefore. The angle of attack increase is only

$$d\alpha = \frac{dU'}{V} \frac{1 - \frac{m}{2} \frac{U}{V}}{\left(1 + \left(\frac{U'}{V}\right)^2\right)}$$

and the equation for m is therefore

$$m = .7 \frac{4S}{D^2} \left(1 - \frac{m}{2} \left(\frac{U}{V}\right)_0\right)$$

giving

$$m = \frac{.7 \frac{4S}{D^2}}{1 + .35 \frac{4S}{D^2} \left(\frac{U}{V}\right)_0}$$

Doctor Durand's propellers with narrow blades have a mean nominal blade width ratio $\frac{2S}{D^2} = 0.15$.

It is actually smaller, say, 0.14, as the portions near the center are inefficient and the tip slightly rounded. $\left(\frac{U}{V}\right)_0$, in most series, changes from about 3 to about 5. The modulus as resulting from the last equation is then 0.15 and 0.13, respectively, giving a mean value of 0.14. The tests give as average $m = 0.133$, which is less than the theoretical value. In view, however, of the fact that the blade sections did not produce quite as much lift as ideal sections do, and that the mean radius 0.7 is chosen a little arbitrarily, the agreement can be considered as good. The blades also twist under the air forces, diminishing the thrust.

The analysis shows thus that the slip curves as obtained from the tests are in substantial agreement with those to be expected from the consideration of the ideal propeller. The explanation offered for propeller action is thus demonstrated to be fundamentally sound, and

the method of using the slip curves for the computation of propellers appears promising. These curves and their constants should be studied for actual propellers and models thereof in order to provide the numerical data necessary to employ successfully the method of the slip curve.

THE TORQUE.

The slip curve represents primarily the relation between the condition of motion of the propeller and the thrust. The designer is even more interested in the torque. Unfortunately, model tests do not lend themselves readily to obtain reliable and exact information on the torque and Doctor Durand's tests even less, because the blade sections are rather unusual. This refers to the determination of the exact torque. Since the greatest portion of the horsepower absorbed by the propeller is transformed into the thrust horsepower, and the efficiency of the propeller is known, at least approximately, there remains no very great doubt about the torque, if the thrust is known; and the exact knowledge of the thrust for a certain condition is the first and chief requirement for the computation of the torque. There remains only some doubt about that portion of the torque which is created by the friction of the blades and by some other minor sources of loss.

The question can be discussed in general at least. A torque coefficient consistent with the thrust coefficient used in this paper is

$$C_q = \frac{Q}{\frac{D}{2} V^2 \frac{\rho}{2} D^2 \frac{\pi}{4}} \text{ (definition)}$$

The torque is divided by half the diameter, by the area of the propeller disk, and by the dynamic pressure of the velocity of flight. This coefficient is the only one which, together with the thrust coefficient C_T and the relative tip velocity U/V , gives a simple expression for the propeller efficiency η without any numerical factor.

$$\eta = \frac{C_T}{C_q} \frac{U}{V}$$

The torque coefficient can be divided into the following three parts:

$$1. \quad \frac{C_T}{U} \frac{1}{V}$$

The horsepower absorbed by this portion is equal to the thrust horsepower.

$$2. \quad \frac{C_T \frac{v}{2V}}{\frac{U}{V}}$$

The horsepower absorbed by this portion is used for building up the theoretical slipstream.

3. The remaining part

$$C_q - \frac{C_T}{U} \left(1 + \frac{v}{2V} \right)$$

comprises all remaining losses, chiefly the friction between the blades and the air. This part of the torque coefficient will therefore assume a more constant value if converted into a sort of drag coefficient of the blades by multiplying it by

$$\frac{D^2 \frac{\pi}{4}}{S \left(\frac{U}{V} \right)^2}$$

I have shown in reference (2) that the drag coefficient so defined and computed is about $C_D = 0.025$ for actual propellers.

COMPUTATION OF THE PITCH.

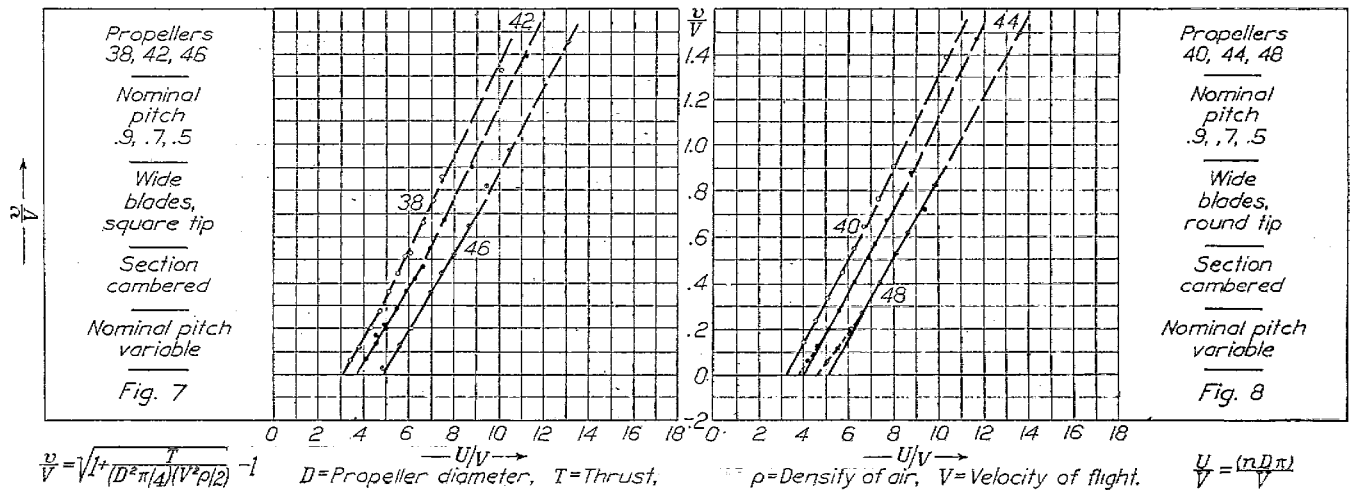
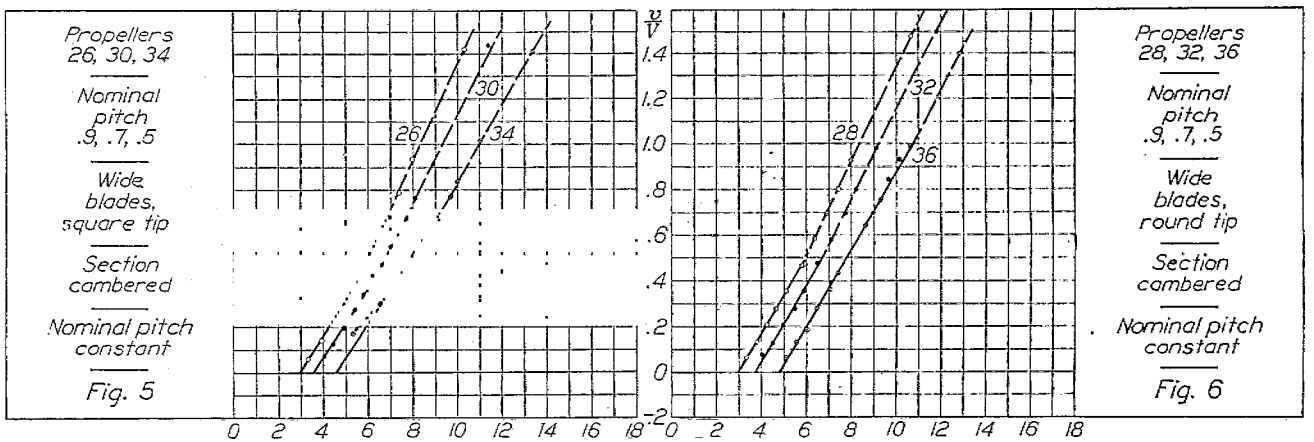
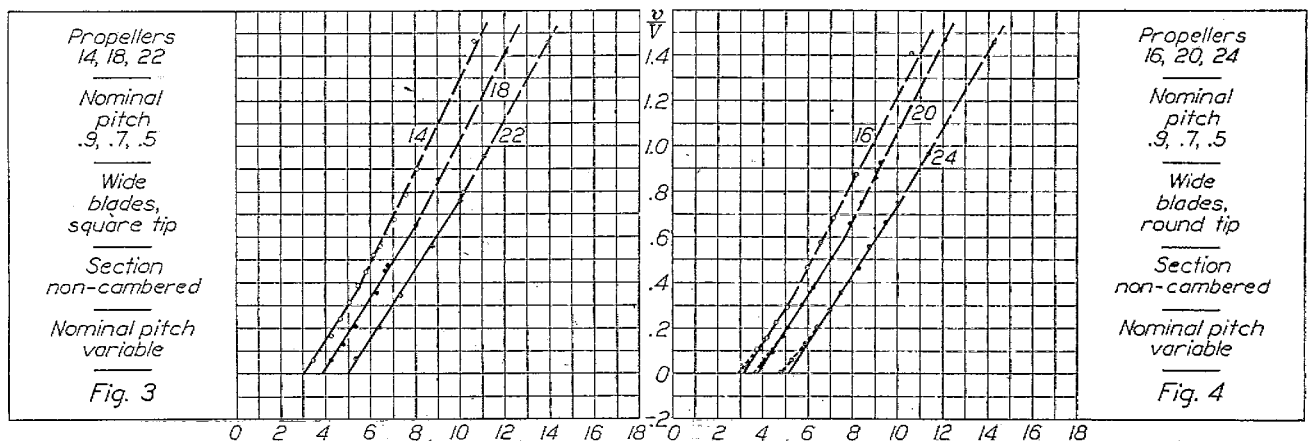
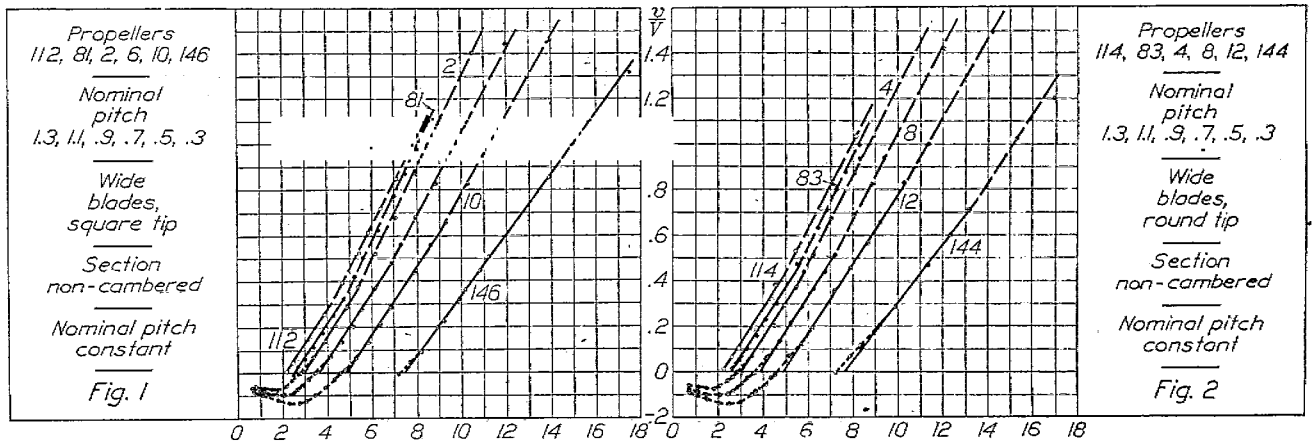
I wish at last to show, in a general way, how to proceed by using the slip curve in the solution of one practical problem often occurring. This is the computation of the pitch. The horsepower, the revolutions per minute, and the velocity of flight may be given. The designer as a rule has no difficulty to decide on the diameter and the blade width of the propeller. He can then estimate the available thrust, and hence the thrust coefficient, and obtains from it the torque coefficient and the torque, using a drag coefficient of the blades in the neighborhood of $C_D = 0.025$. The torque has to check with the available horsepower, otherwise the calculation has to be repeated. The designer knows now the thrust coefficient and the revolutions, and computes from the former the relative slip velocity and from the latter the relative tip velocity, using equations (4) and (5). The slip modulus m is known for the type of propeller used and its blade width. It is not very different from $m = 1/8$ for ordinary propellers. That gives the relative tip velocity for the thrust zero.

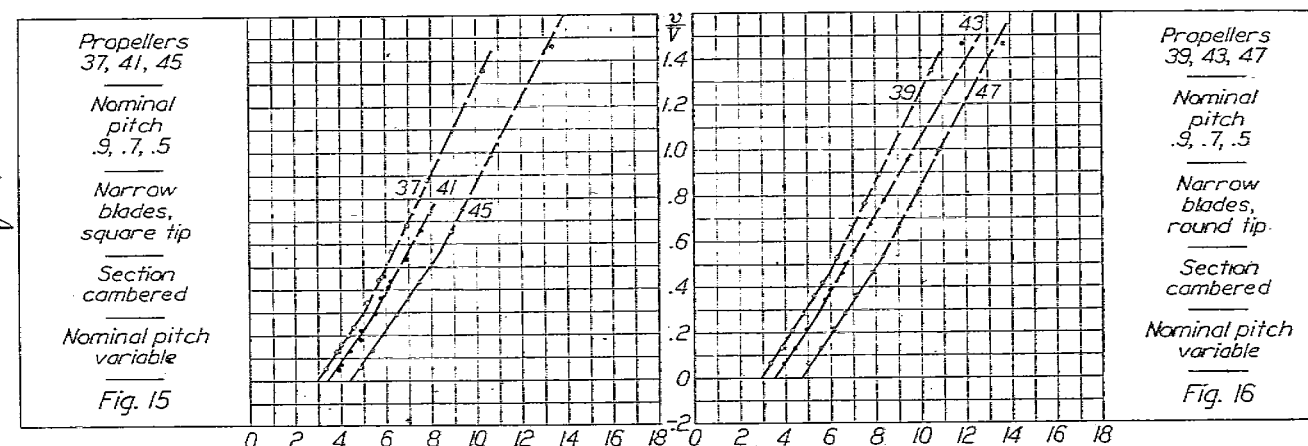
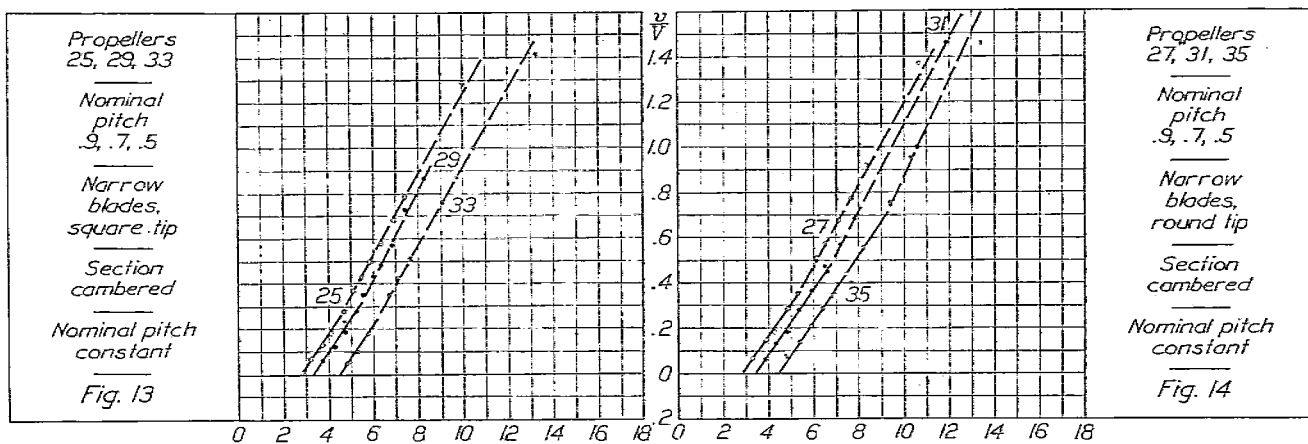
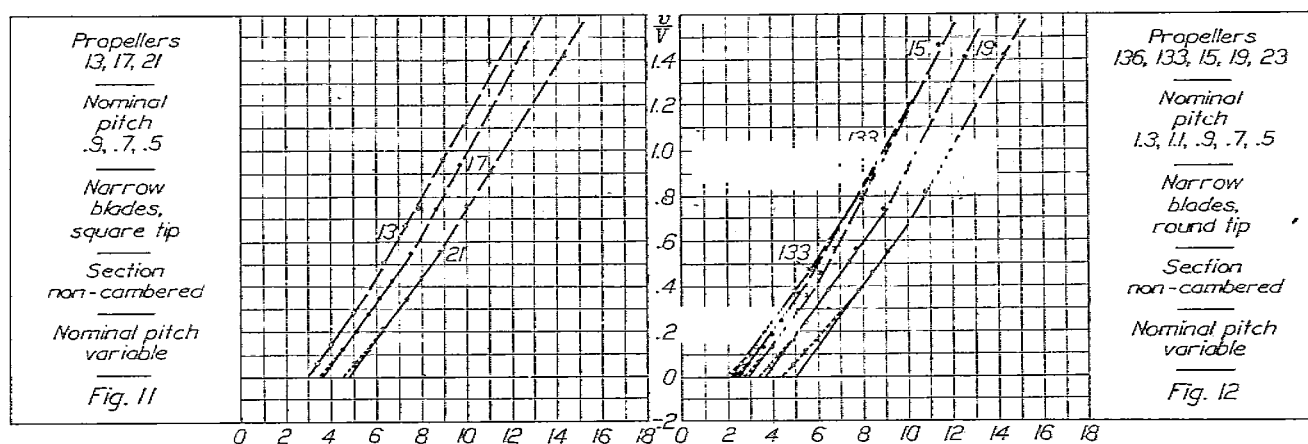
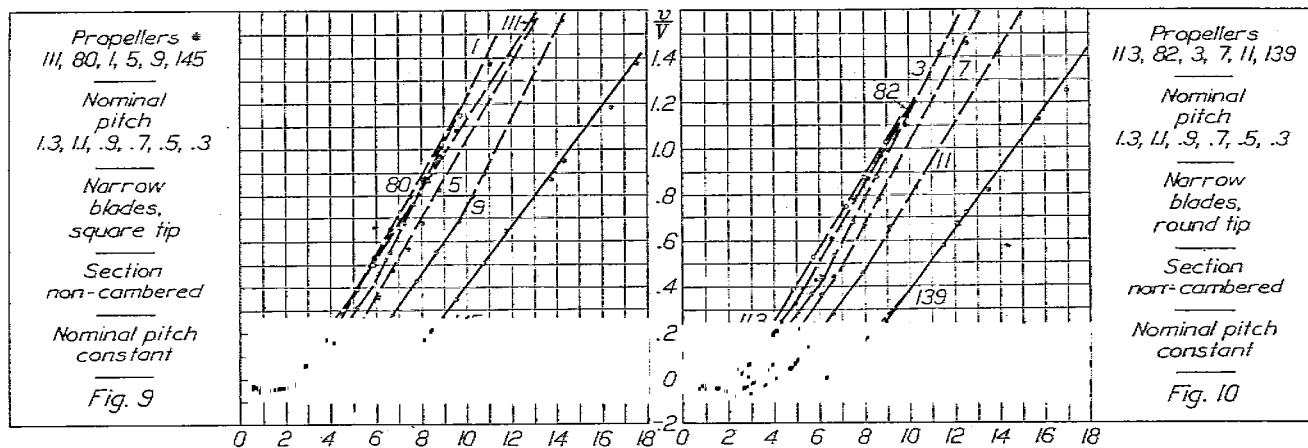
$$\left(\frac{U}{V}\right)_0 = \frac{U}{V} - \frac{v}{mV}$$

and the effective pitch is then

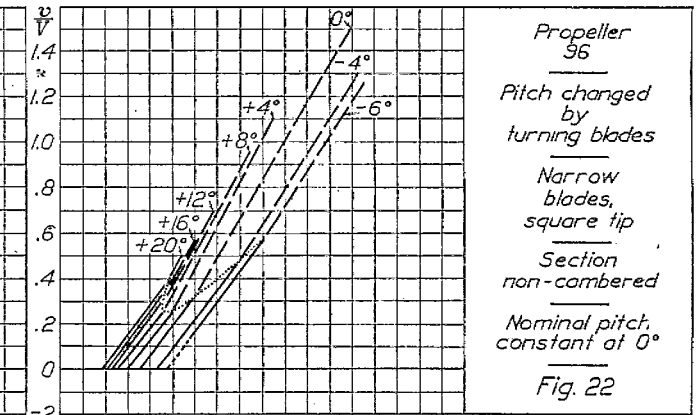
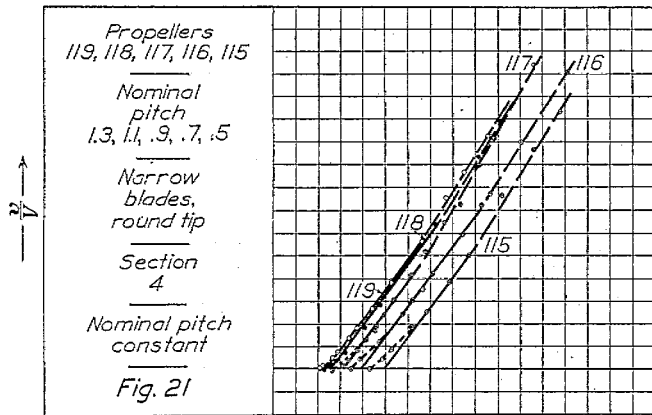
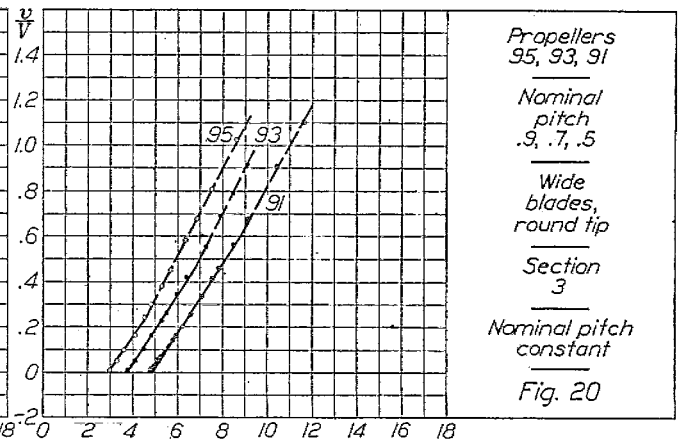
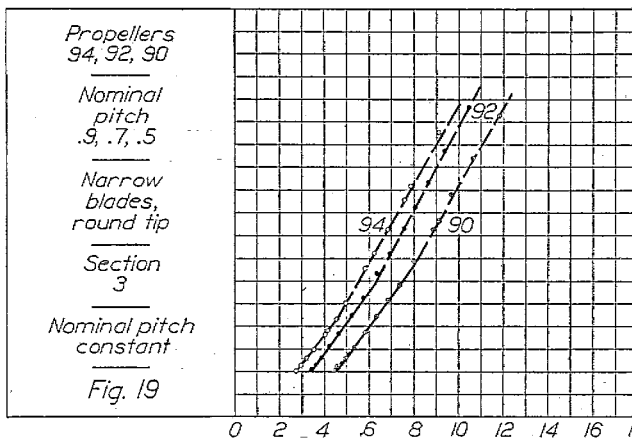
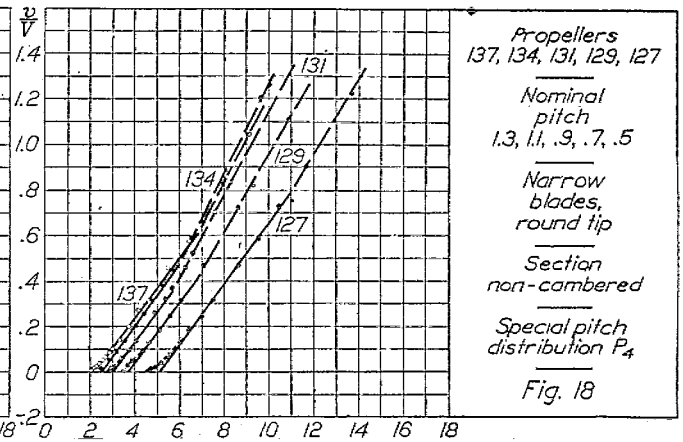
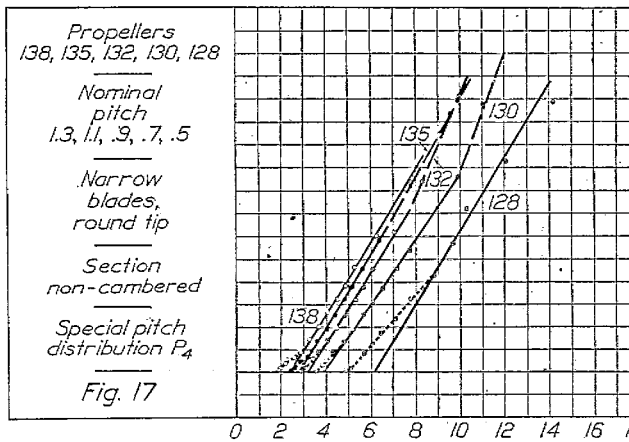
$$p_e = \frac{D\pi}{V}$$

The nominal pitch is smaller, as explained above. It may be, however, that the difference is not large, as the elastic twist of the blades may neutralize their camber effect. This has to be determined in flight.





$$\frac{V}{V_0} = \frac{U}{V_0} \sqrt{1 + \frac{T}{\rho V_0^2 A}} \quad \text{--- } U/V \text{ ---} \quad \text{--- } U/V \text{ ---} \quad \frac{U}{V_0} = \frac{(nD)\pi}{V_0}$$



$$\frac{V}{V} = \sqrt{1 + \frac{T}{(D^2 \pi/4)(V^2 \rho/2)}} - 1$$

D = Propeller diameter, T = Thrust.

$$\frac{U}{V} = \frac{(n D \pi)}{V}$$

ρ = Density of air, V = Velocity of flight.